NCERT Class 10 Chapter 5 Quadratic Equations Official CBSE Board Sample Problems Short Answer (For CBSE, ICSE, IAS, NET, NRA 2022)

Question

If- 5 is a root of the quadratic equation $2x^2 + px - 15 = 0$ and the quadratic equation $p\left(x^2 + x\right) + k = 0$ has equal roots, find the value of k

Solution

$\because$— 5 is the root of the quadratic equation $2x^2 + px - 15 = 0$

$\Rightarrow 2(-5)^2 + p(-5) - 15 = 0$

$\Rightarrow 50 - 5p - 15 = 0 \Rightarrow 35 - 5p = 0$

$\Rightarrow 5p = 35 \Rightarrow p = 7$

Now, given that equation $p\left(x^2 + x\right) + k = 0$ has equal roots

i.e.. $7\left(x^2 + x\right) + k = 0$ has equal roots

i.e.. $+7x + k = 0$ has equal roots

$\Rightarrow 7^2 - 4 \times 7 \times k = 0$ ($\because$ For equal roots, D = *o, i.e.. b^2*—4ac = 0]

$\Rightarrow 7(7 - 4k) = 0$

$\Rightarrow k = \frac{7}{4}$

Question

If $x = \frac{2}{3}$ and $x = -3$ are roots of the quadratic equations $ax^2 + 7x + b = 0$, find the values ofaand b.

Solution

Given quadratic equation is $ax^2 + 7x + b = 0$ . . (i)

$a\left(\frac{2}{3}\right)^2 + 7\left(\frac{2}{3}\right) + b = 0$ ($\because$ x = is the root of equation (i))

$\Rightarrow \frac{4}{9}a + \frac{14}{3} + b = 0$

$\Rightarrow \frac{4a + 42 + 9b}{9} = 0 \Rightarrow 4a + 9b + 42 = 0$ ... (ii)

$\Rightarrow a(-3)^2 + 7(-3) + b = 0$ [$\because x = -3$ is the root of equation (i)]

$\Rightarrow 9a + b - 21 = 0$ .. *(iii)*

Putting the value of b from (iii) in *(ii)* , we get

$\Rightarrow 4a + 9[21 - 9a] + 42 = 0$

$\Rightarrow 4a + 189 - 81a + 42 = 0$

$\Rightarrow 231 - 77a = 0$

$\Rightarrow 77a = 231$

$\Rightarrow a = 3$

Putting a = 3 in *(iii)* , we have

$27 + b = 21$

$\Rightarrow b = -6$

$a = 3, b = -6$

Question

A two digit number is four times the sum of the digits. It is also equal to 3 times the product of digits. Find the number.

Solution

Let ones digit of number = x

Let tens digit of number = y

$\therefore$ Number will be $= 10y + x$

According to question,

$\Rightarrow 10y + x = 4(x + y)$

$\Rightarrow 10y + x = 4x + 4y$

$3x - 6y = 0$

$x - 2y = 0 \Rightarrow x = 2y$ ... (i)

and $10y + x = 3$ xy ... (ii)

Putting $x = 2y$ from (i) in *(ii)* , we get,

$10y + 2y = 3(2y)\, y \Rightarrow 12y = 6y^2 \Rightarrow y = 2$

and $x = 2 \Rightarrow x = 4 (\because y = 2)$

The required number $= 10(2) + 4 = 20 + 4 = 24$.

Question

$$\frac{1}{(x-1)(x-2)} = \frac{1}{(x-2)(x-3)} = \frac{2}{3}, x \neq 1,2,3$$

Solve for x:

Solution

$$\frac{1}{(x-1)(x-2)} = \frac{1}{(x-2)(x-3)} = \frac{2}{3}, x \neq 1,2,3$$

$$\Rightarrow \frac{x-3+x-1}{(x-1)(x-2)(x-3)} = \frac{2}{3} \Rightarrow \frac{(2x-4)}{(x-1)(x-2)(x-3)} = \frac{2}{3}$$

$$\Rightarrow \frac{2(x-2)}{(x-1)(x-2)(x-3)} = \frac{2}{3}$$

$$\Rightarrow 3 = (x-1)(x-3) \Rightarrow x^2 - 4x + 3 = 3$$

$$\Rightarrow x^2 - 4x = 0 \Rightarrow x(x-4) = 0 \Rightarrow x = 0 \text{ or } x = 4$$

Question

If the roots of the quadratic equation $(a-b)x^2 + (b-c)x + (c-a) = 0$ **are equal, prove that** $2a = b + c$.

Solution

$$(a-b)x^2 + (b-c)x + (c-a) = 0$$

For equal roots, discriminant, D = o

$$\Rightarrow (b-c)^2 - 4(a-b)(c-a)\,0$$

$$\Rightarrow b^2 - 2bc + c^2 - 4(ac - a2 - bc + ab) = 0$$

$$\Rightarrow b^2 - 2bc + c^2 - 4ac + 4a^2 + 4bc - 4ab = 0$$

$$\Rightarrow 4a^2 + b^2 + c^2 - 4ab + 2bc - 4ac = 0$$

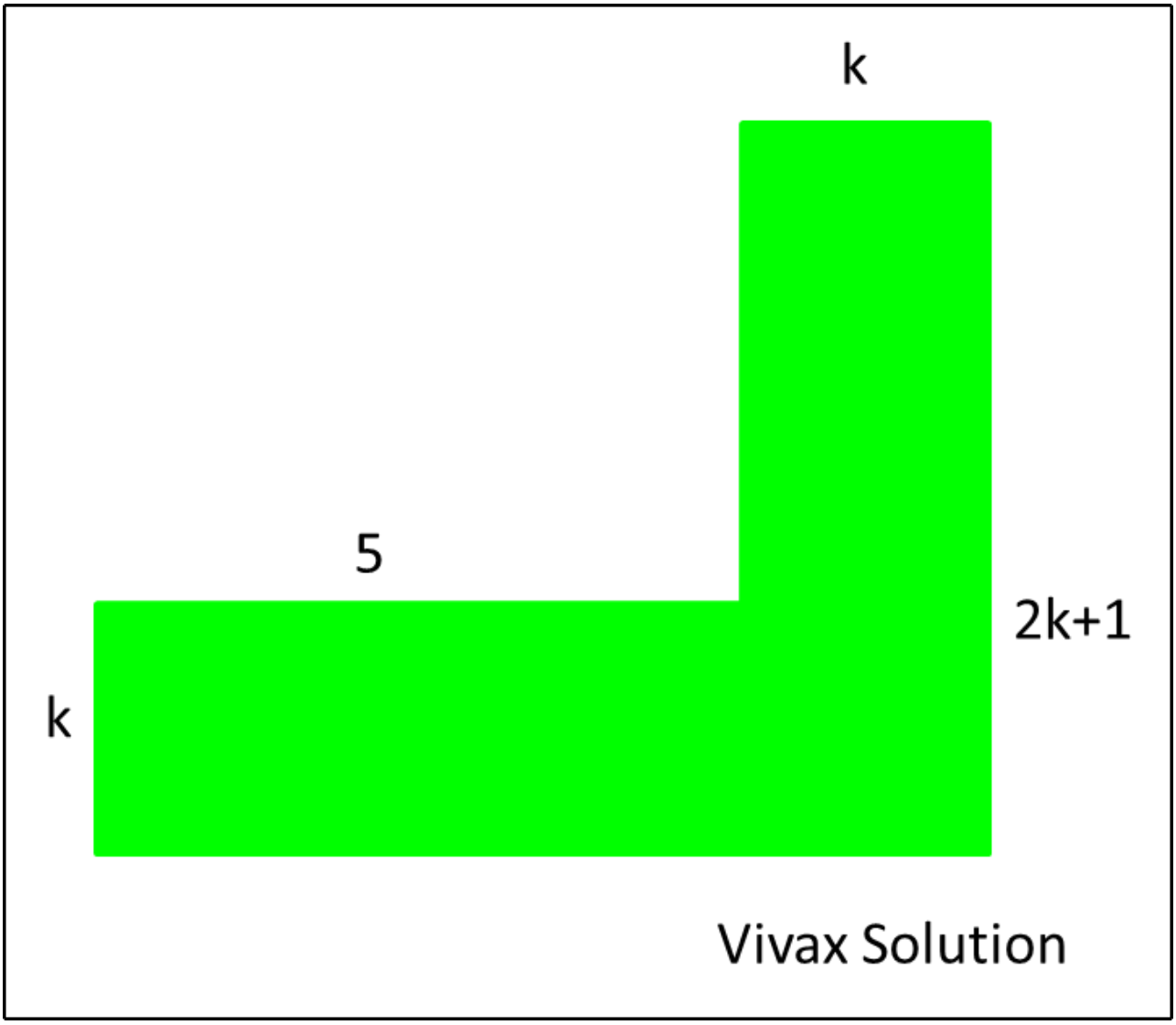
$$\Rightarrow (2a - b - c)^2 = 0$$

$$\Rightarrow 2a - b - c = 0$$

$$\Rightarrow 2a = b + c$$

Question

The following picture shows the shape of a certain grass patch. If the area of the patch is $80m^2$ **, find k.**



Solution

The total area $= 5k + k\,(2k + 1)$

$= 5k + 2k^2 + k$

$= 2k^2 + 6k$

Since the area is $80m^2$

$$2k^2 + 6k = 80$$

$$2k^2 + 6k - 80 = 0$$

$$(2k - 10)(k + 8) = 0$$

$$k = 5 \text{ork} = -8$$

Since the length cannot be negative, $k = 5$.

Question

Using the quadratic formula, solve the quadratic equation: $x - \frac{31}{x} = 0$

Solution

$-\sqrt{31}$ *and* $\sqrt{31}$

$$x - \frac{31}{x} = 0$$

$$x^2 - \frac{31}{x} = 0$$

$$x^2 - 31 = 0$$

$$x = 0 \pm \sqrt{\frac{-4.(31)}{2}}$$

$$= \pm 2\sqrt{\frac{(31)}{2}}$$

$$= \pm\sqrt{31}$$

Question

Find 2 numbers that sum to 21 and the sum of the squares is 261.

Solution

The numbers are 15 and 6.

$$x + y = 21 \Rightarrow x = 21 - y$$

$$x^2 + y^2 = 261$$

$$(21 - y)^2 + y^2 = 261$$

$$441 - 42y + y^2 + y^2 = 261$$

$$2y^2 - 42y + 180 = 0$$

$$y^2 - 21y \mp 90 = 0$$

$$y^2 - 15y - +90 = 0$$

$$y_1 = 15$$

$$y_2 = 6$$

Hence, $x_1 = 6$ and $x_2 = 15$

Question

Find the roots of the following equations:

$$x - \frac{3}{x+3} - x + \frac{3}{x-3} = 6\frac{6}{7}, (x \neq 3, -3)$$

Solution

On simplifying we get

$$4x^2 + 7x - 36 = 0$$

$$(x+4)(4x-9) = 0$$

$$x = -4, \frac{9}{4}$$

Question

If p and q are the roots of the equation $x^2 - px + q = 0$, then find the value of p and q.

Solution

Since p and q are the roots of this equation, Hence: $p^2 - p^2 + q = 0$. Hence, $q = 0$

$$q^2 + pq + q = 0$$

$$q(q - p + 1) = 0$$

$$q = 0 or p = q + 1$$

Hence, $p = 1$

Question

Find the value of k for which the quadratic equation $4x^2 - 3kx + 1 = 0$ has two real and equal roots.

Solution

For equal roots $D = 0$

$$\Rightarrow (-3k)^2 - 4(4)(1) = 0$$

$$9k^2 - 16 = 0$$

$$\Rightarrow k = \pm\frac{4}{3}$$

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