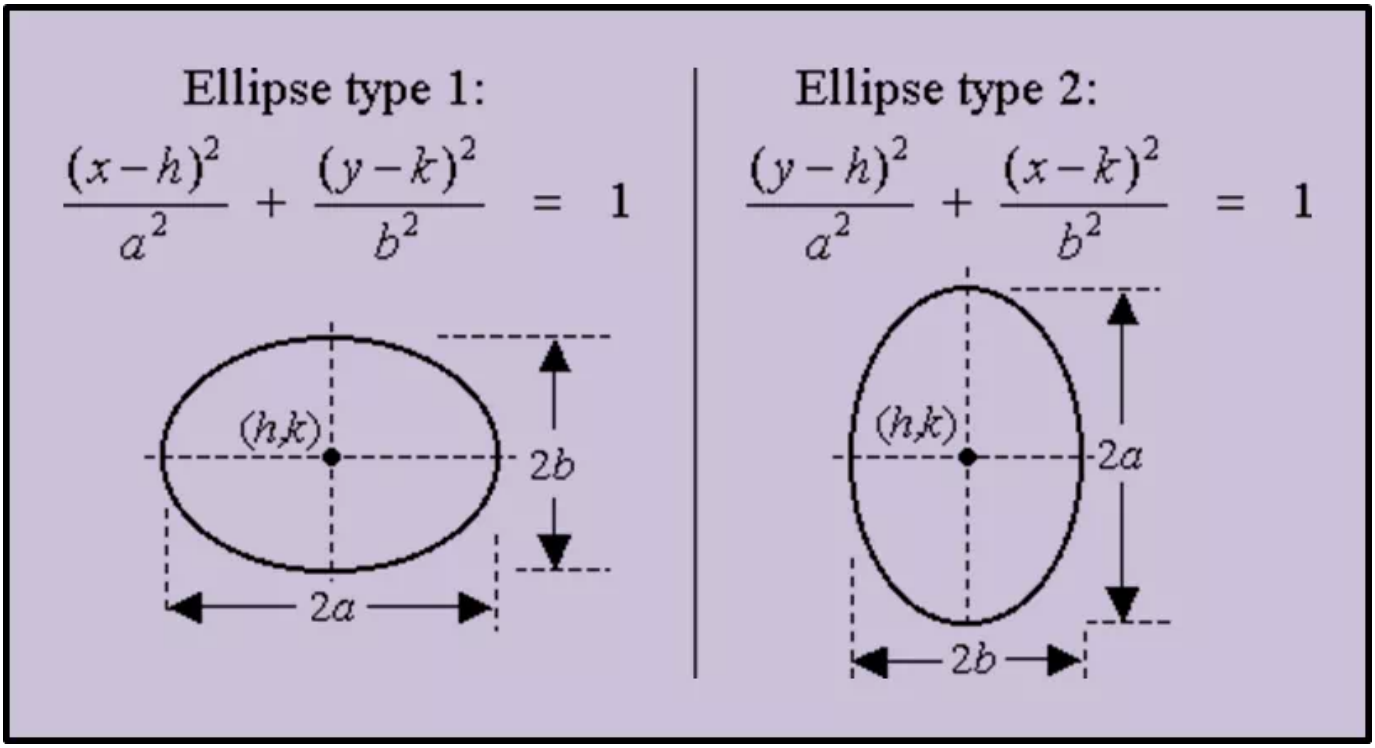
Flexiprep

Examrace 339K

NCERT Class 11 Mathematics Solutions: Chapter 11 –Conic Sections Miscellaneous Exercise 11 Part 4



Formula of Ellipse

1. Find the area of the triangle formed by the lines joining the vertex of the parabola $x^2 = 12y$ to the ends of its latus rectum.

Answer:

The given parabola is $x^2 = 12y$.

On comparing this equation with $x^2 = 4ay$,

we obtain

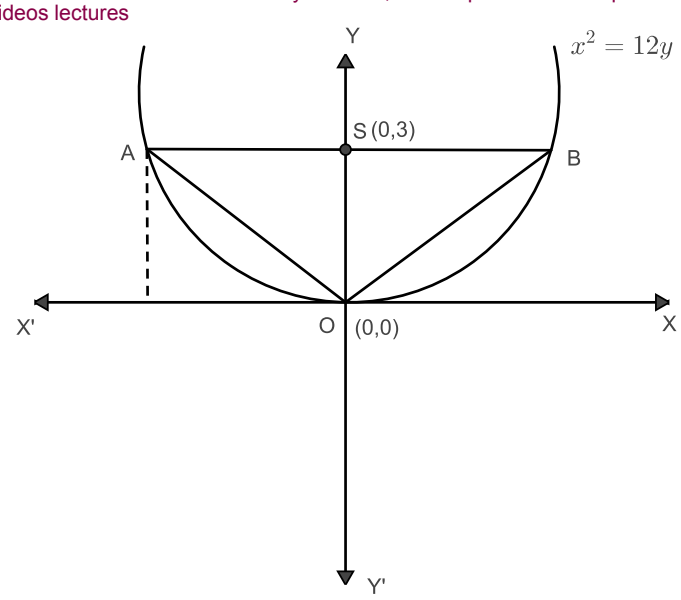
$$4a = 12$$

$$\Rightarrow a = 3$$

$\therefore$The coordinates of foci are $S(0, a) = S(0, 3)$

Consider AB be the latus rectum of the given parabola.

The given parabola can be roughly drawn as



The Given Parabola

At $y = 3$,

$$x^2 = 12\ (3)$$

$$\Rightarrow x^2 = 36$$

$$\Rightarrow\ x\ =\ \pm 6$$

$\therefore$ The coordinates of A *are* $(-6,\ 3)$, while the coordinates of B *are* $(6,\ 3)$.

So, the vertices of ΔOAB are $O\ (0,\ 0)$, $A\ (-6,\ 3)$ *and* $B\ (6,\ 3)$.

Area of $\Delta OAB = \frac{1}{2}\,|0\,(3-3) + (-6)\,(3-0) + 6(0-3)|\ unit^2$

$$= \frac{1}{2}\,|(-6)\,(3) + 6(-3)|\ unit^2$$

$$= \frac{1}{2}\,|-18-18|\ unit^2$$

$$= \frac{1}{2}\,|-36|\ unit^2$$

$$= \frac{1}{2} \times 36\ unit^2$$

$$= 18\ unit^2$$

So, the required area of the triangle is $18\ unit^2$

2. A man running a racecourse notes that the sum of the distances from the two flag posts form him is always $10\ m$ and the distance between the flag posts is $8\ m$. find the equation of the posts traced by the man.

Answer:

Consider A *and* B be the positions of the two flag posts and $P\,(x,\ y)$ be the position of the man.

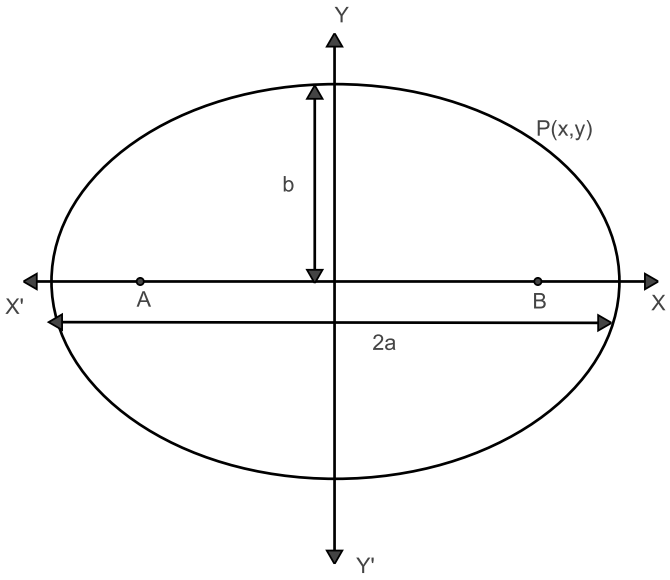
Accordingly,

$$PA + PB = 10$$

We know that if a point moves in a plane in such a way that the sum of its distances from two fixed points is constant, then the path is an ellipse and this constant value is equal to the length of the major axis of the ellipse.

So, the path described by the man is an ellipse where the length of the major axis is $10\,m$, while points A *and* B are the foci.

Taking the origin of the coordinate plane as the centre of the ellipse, while taking the major axis along the $x-$ axis, the ellipse can be diagrammatically represented as,



The Ellipse Can Be Diagrammatically Represented.

The equation of the ellipse will be of the form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Accordingly, $2a = 10$

$$\Rightarrow a = 5$$

Distance between the foci

$$(2c) = 8$$

$$\Rightarrow c = 4$$

On using the relation

$$c = \sqrt{a^2 - b^2}$$

$$\Rightarrow 4 = \sqrt{25 - b^2}$$

$$\Rightarrow 16 = 25 - b^2$$

$$\Rightarrow b^2 = 25 - 16 = 9$$

$\Rightarrow b = 3$

So, the equation of the path traced by the man is $\frac{x^2}{25} + \frac{y^2}{9} = 1$