FlexiPrep

Linear Equations: Definition and Standard Form of Linear Equation

Linear equations are those equations that are of the first order. These equations are defined for lines in the coordinate system.

Linear equations are also first-degree equations as it has the highest exponent of variables as 1. Some of the examples of such equations are as follows:

- $2x - 3 = 0,$
- $2y = 8$
- $m + 1 = 0,$
- $\frac{x}{2} = 3$
- $x + y = 2$
- $3x - y + z = 3$

When the equation has a homogeneous variable (i.e. only one variable), then this type of equation is known as a **Linear equation in one variable**. Line equation is achieved by relating to zero a linear polynomial over any field, from which the coefficients are obtained.

The solutions of linear equations will generate values, which when substituted for the unknown values, make the equation true. In the case of one variable, there is only one solution, such as x+2=0. But in case of the two-variable linear equation, the solutions are calculated as the Cartesian coordinates of a point of the Euclidean plane.

Linear Equation Definition

What is a linear equation definition and example? An Equation having the maximum order of 1 is known as a Linear equation.

Below are some examples of linear equations in 1 variable, 2 variables and 3 variables:

Linear Equation in one variable	Linear equation in Two Variable	Linear Equation in Three variable
$3x + 5 = 0$	$y + 7x = 3$	$x + y + z = 0$

$\frac{3}{2}\boldsymbol{x} + 7 = 0$	$3\boldsymbol{a} + 2\boldsymbol{b} = 5$	$\boldsymbol{a} - 3\boldsymbol{b} = \boldsymbol{c}$
$98\boldsymbol{x} = 49$	$6\boldsymbol{x} + 9\boldsymbol{y} - 12 = 0$	$3\boldsymbol{x} + 12\boldsymbol{y} = \frac{1}{2}\boldsymbol{z}$

Linear Equation Definition

Linear Equations Formula

There are different forms to write linear equations. Some of them are:

Linear Equation	General Form	Example
Slope intercept form	$y = mx + c$	$y + 2x = 3$
Point – slope form	$y = y_1 = m(x - x_1)$	$y - 3 = 6(x - 2)$
General Form	$Ax + By + C = 0$	$2x + 3y - 6 = 0$
Intercept Form	$\frac{x}{x_0} + \frac{y}{y_0} = 1$	$\frac{x}{2} + \frac{y}{3} = 1$
As a Function	$f(x)$ instead of y $f(x) = x + c$	$f(x) = x + 3$
The identity Function	$f(x) = x$	$f(x) = 3x$
Constant Function	$f(x) = C$	$f(x) = 6$

Linear Equations Formula

Where m = slope of a line; (x_0, y_0) intercept of x-axis and y-axis.

Equation of a Line

There are many forms through which a line is defined in an X-Y plane. Some of the common forms used here for solving linear equations are:

- General Form
- Slope Intercept Form
- Point Form
- Intercept Form

- Two-Point form

Standard Form of Linear Equation

Linear equations are a combination of constants and variables. The standard form of a linear equation in one variable is represented as $ax + b = 0$ where, $a \neq 0$ and x is the variable.The standard form of a linear equation in two variables is represented as

$$ax + by + c = 0,\ where\ a \neq 0,\ b \neq 0,\ x\ and\ y\ are\ the\ variables.$$

The standard form of a linear equation in three variables is represented as

$$ax + by + cz + d = 0\ where\ a \neq 0,\ b \neq 0,\ c \neq 0,\ x, y, z\ are\ the\ variables.$$

Slope Intercept Form

The most common form of linear equations is in slope-intercept form, which is represented as,

$$y = mx + c$$

where y and x are the point in x-y plane, m is the slope of the line (also called gradient) and c is the intercept (a constant value).

For example, $y = 3x + 7$

slope, $m = 3$ and intercept $= 7$

Point Slope Form

In this form of linear equation, a straight-line equation is formed by considering the points in x-y plane, such that:

$$y – y_1 = m(x – x_1)$$

where (x_1, y_1) are the coordinates of the line.

We can also express it as:

$$y = mx + y_1 - mx_1$$

Intercept Form

A line which neither parallel to x-axis or y-axis nor it pass through the origin but intersects the axes in two different points. The intercept values x_o and y_o of these two points are nonzero and forms an equation of the line is:

$$\frac{x}{x_0} + \frac{y}{y_0} = 1$$

Two-Point Form

If there are two points say, (x_1, y_1) and (x_2, y_2) and only one line passes through them, then the equation of the line is given by:

$$y - y_1 = \left[(y_2 - y_1)/(x_2 - x_1)\right](x - x_1)$$

where $(y_2 - y_1)/(x_2 - x_1)$ is the slope of the line and $x_1 \neq x_2$

How to Solve Linear Equations

Let's solve such linear or line equations in one variable, two variables as well as in three variables with examples. Solving these equations with step by step procedure are given here.

Solution of Linear Equations in One Variable

Both sides of the equation are supposed to be balanced for solving a linear equation. Equality sign denotes that the expressions on either side of the 'equal to' sign are equal. Since the equation is balanced, for solving it certain **mathematical operations** are performed on both sides of the equation in a manner that it does not affect the balance of the equation. Here is the example related to the linear equation in one variable.

Example: Solve $2\boldsymbol{x} - \frac{10}{2} = 3(\boldsymbol{x} - 1)$

Clear the fraction

$$2x - 5 = 3(x - 1)$$

Simplify Both sides equations

$$2x - 5 = 3x - 3$$

$$2x = 3x + 2$$

$$2x - 3x = 2$$

Isolate x

$$x = -2$$

Solution of Linear Equations in Two Variables

To solve Linear Equations having 2 variables, there are different methods. Following are some of them:

- Method of substitution
- Cross multiplication method
- Method of elimination
- Determinant methods

We must choose a set of 2 equations to find the values of 2 variables. Such as ax + by + c = 0 and $dx + ey + f = 0$, also called a system of equations with two variables, where x and y are two variables and a, b, c, d, e, f are constants, and $a, b, d \text{ and } e$ are not zero. Else, the single equation has an infinite number of solutions.

Solution of Linear Equations in Three Variables

To solve Linear Equations having 3 variables, we need a set of 3 equations as given below to find the values of unknowns. Matrix method is one of the popular methods to solve system of linear equations with 3 variables.

$$a_1 x + b_1 y + c_1 z + d_1 = 0$$

$$a_2 x + b_2 y + c_2 z + d_2 = 0 \text{ and}$$

$$a_3 x + b_3 y + c_3 z + d_3 = 0$$

Linear Equations Problems and Solutions

Example 1: Solve $x = 12(x + 2)$

Solution:

$$x = 12(x + 2)$$

$$x = 12x + 24$$

Subtract 24 from each side

$$x - 24 = 12x + 24 - 24$$

$$x - 24 = 12x$$

Simplify

$$11x = -24$$

Isolate x, by dividing each side by 11

$$\frac{11x}{11} = -\frac{24}{11}$$

$$x = -\frac{24}{11}$$

Example 2: Solve $x - y = 12$ and $2x + y = 22$

Solution:

Name the equations

$x - y = 12$ ———- (1)

$2x + y = 22$ ———- (2)

Isolate Equation (1) for x,

$$x = y + 12$$

Substitute y + 12 for x in equation (2)

$$2\left(y+12\right)+y=22$$

$$3y\ +\ 24\ =\ 22$$

$$3y\ =\ -2$$

or $y\ =\ -\frac{2}{3}$

Substitute the value of y in $x\ =\ y\ +\ 12$

$$x\ =\ y\ +\ 12$$

$$x\ =\ -2/3\ +\ 12$$

$$x\ =\frac{34}{3}$$

Answer: $x\ =\ \frac{34}{3}$ and $y\ =\ -\frac{2}{3}$

Question: 3 Solve $\frac{[17\ (2-y)-5(y+12)]}{1-7y}=8$

Solution:

$$\frac{[17\ \left(2-y\right)-5\left(y+12\right)]}{1-7y}=8$$

Open all bracket,

$$34-17y-5y-60=8\left(1-7y\right)$$

$$34-17y-5y-60=8-56y$$

$$-22y-26=8-56y$$

$$-22y+56y=8+26$$

$$34y=34$$

$$y=\frac{34}{34}$$

$$y=1$$