

FlexiPrep

Solving Linear Equations: Graphical Method of Solving Linear Equations

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Solving linear equations means to find the solution of a linear equation like linear equations in one variable, linear equations in two variables and linear equations in three variables.

What Does Solving Linear Equations Mean?

Solving a linear equation refers to finding the solution of linear equations in one, two, three or variables. In simple words, a solution of a linear equation means the value or values of the variables involved in the equation.

How to Solve Linear Equations?

There are six main methods to solve linear equations. These methods for finding the solution of linear equations are:

- Graphical Method
- Elimination Method
- Substitution Method
- Cross Multiplication Method
- Matrix Method
- Determinants Method

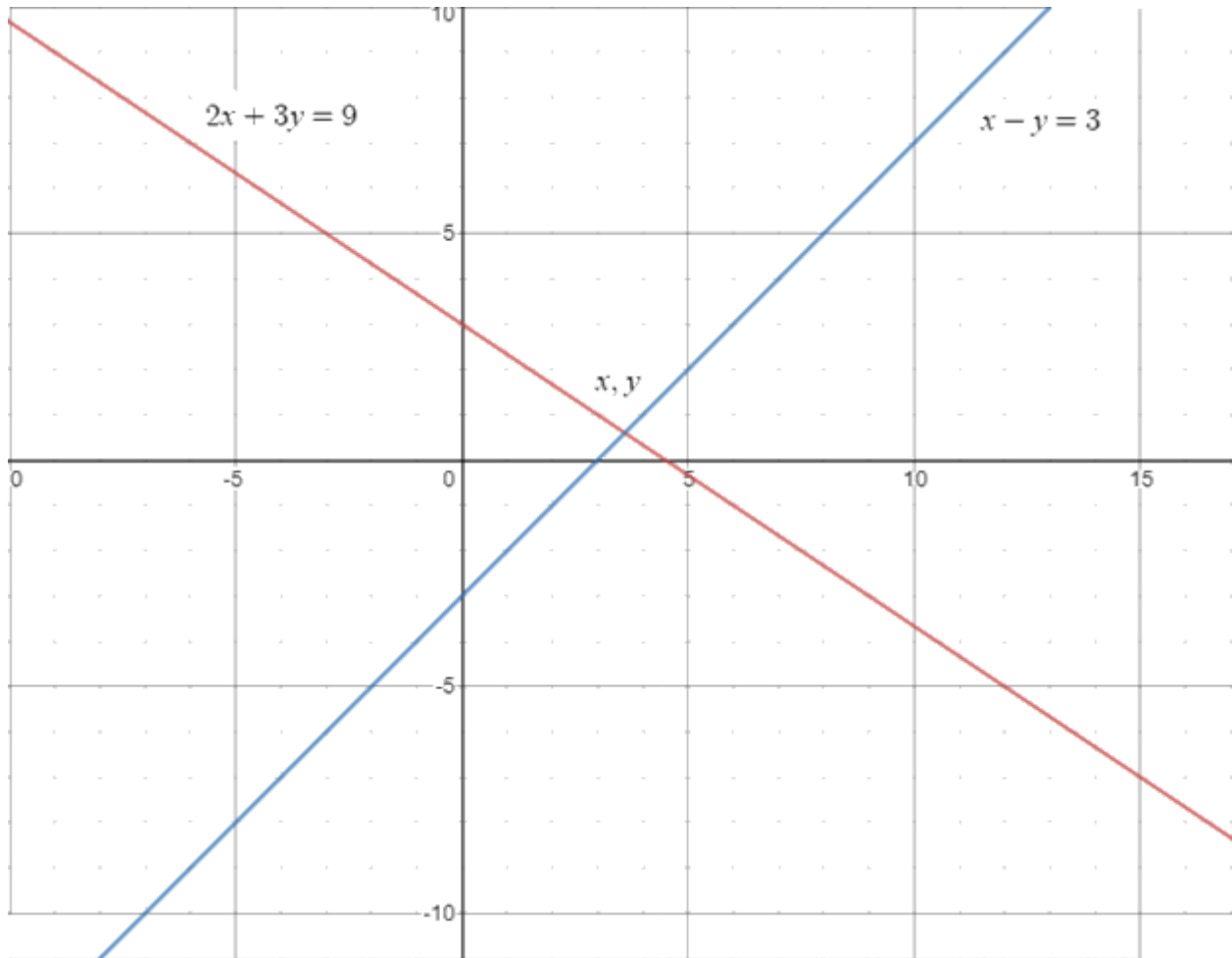
Graphical Method of Solving Linear Equations

To solve linear equations graphically, first graph both equations in the same coordinate system and check for the intersection point in the graph. For example, take two equations as $2x + 3y = 9$ and $x - y = 3$.

Now, to plot the graph, consider $x = 0, 1, 2, 3, 4$ and solve for y . Once (x, y) is obtained, plot the points on the graph. It should be noted that by having more values of x and y will make the graph more accurate.

Check: Graphical Method of Solving Linear Programming

The graph of $2x + 3y = 9$ and $x - y = 3$ will be as follows:



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In the graph, check for the intersection point of both the lines. Here, it is mentioned as (x, y) . Check the value of that point and that will be the **solution of both the given equations**. Here, the value of $(x, y) = (3.6, 0.6)$.

Elimination Method of Solving Linear Equations

In the elimination method, any of the coefficients is first equated and eliminated. After elimination, the equations are solved to obtain the other equation. Below is an example of solving linear equations using the elimination method for better understanding.

Consider the same equations as

$$2x + 3y = 9 \text{ --- (i)}$$

And,

$$x - y = 3 \text{ --- (ii)}$$

Here, if equation (ii) is multiplied by 2, the coefficient of "x" will become the same and can be subtracted.

So, multiply equation (ii) $\times 2$ and then subtract equation (i)

$$2x + 3y = 9$$

$$\frac{2x-2y=6}{-5y=-3}$$

$$\text{Or, } y = \frac{3}{5} = 0.6$$

Now, put the value of $y = 0.6$ in equation (ii).

$$\text{So, } x - 0.6 = 3$$

$$\text{Thus, } x = 3.6$$

In this way, the value of x , y is found to be 3.6 and 0.6.

Substitution Method of Solving Linear Equations

To solve a linear equation using the substitution method, first, isolate the value of one variable from any of the equations. Then, substitute the value of the isolated variable in the second equation and solve it. Take the same equations again for example.

Consider,

$$2x + 3y = 9 \text{--- (i)}$$

And,

$$x - y = 3 \text{--- (ii)}$$

Now, consider equation (ii) and isolate the variable "x".

So, equation (ii) becomes,

$$x = 3 + y.$$

Now, substitute the value of x in equation (i). So, equation (i) will be-

$$2x + 3y = 9$$

$$2(3 + y) + 3y = 9$$

$$6 + 2y + 3y = 9$$

$$\text{Or, } y = \frac{3}{5} = 0.6$$

Now, substitute "y" value in equation (ii).

$$x - y = 3$$

$$\Rightarrow x = 3 + 0.6$$

$$\text{Or, } x = 3.6$$

Thus, $(x, y) = (3.6, 0.6)$.

Cross Multiplication Method of Solving Linear Equations

Linear equations can be easily solved using the cross-multiplication method. In this method, the cross-multiplication technique is used to simplify the solution. For the cross-multiplication method for solving 2 variable equation, the formula used is:

$$\frac{x}{b_1 c_2 - b_2 c_1} = \frac{y}{c_1 a_2 - c_2 a_1} = \frac{1}{b_2 a_1 - b_1 a_2}$$

For example, consider the equations

$$2x + 3y = 9 \text{--- (i)}$$

And,

$$x - y = 3 \text{--- (ii)}$$

Here,

$$a_1 = 2, b_1 = 3, c_1 = -9$$

$$a_2 = 1, b_2 = -1, c_2 = -3$$

Now, solve using the aforementioned formula.

$$x = \frac{b_1 c_2 - b_2 c_1}{b_2 a_1 - b_1 a_2}$$

Putting the respective value, we get,

$$x = \frac{18}{5} = 3.6$$

Similarly, solve for y.

$$y = (c_1 a_2 - c_2 a_1) / (b_2 a_1 - b_1 a_2)$$

$$\text{So, } y = \frac{3}{5} = 0.6$$

Matrix Method of Solving Linear Equations

Linear equations can also be solved using matrix method. This method is extremely helpful for solving linear equations in two or three variables. Consider three equations as:

$$a_1 x + a_2 y + a_3 z = d_1$$

$$b_1 x + b_2 y + b_3 z = d_2$$

$$c_1 x + c_2 y + c_3 z = d_3$$

These equations can be written as:

$$\begin{bmatrix} a_1x + a_2y + a_3z \\ b_1x + b_2y + b_3z \\ c_1x + c_2y + c_3z \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix} \Rightarrow \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

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$$AX = B \text{--- (i)}$$

Here, the A matrix, B matrix and X matrix are:

$$A = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

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Now, multiply (i) by A^{-1} to get:

$$A^{-1}AX = A^{-1}B \Rightarrow I.X = A^{-1}B$$

$$X = A^{-1}B$$

- Learn more on how to solve linear equations with matrix method here.

Determinant Method of Solving Linear Equations (Cramer's Rule)

Determinants method can be used to solve linear equations in two or three variables easily. For two variables and three variables of linear equations, the procedure is as follows.

For Linear Equations in Two Variables:

$$x = \frac{\Delta_1}{\Delta},$$

$$y = \frac{\Delta_2}{\Delta}$$

$$\text{Or, } x = \frac{b_1 c_2 - b_2 c_1}{b_2 a_1 - b_1 a_2} \text{ and } y = \frac{c_1 a_2 - c_2 a_1}{b_2 a_1 - b_1 a_2}$$

Here,

$$\Delta_1 = \begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix}, \Delta_2 = \begin{vmatrix} c_1 & a_1 \\ c_2 & a_2 \end{vmatrix} \text{ and } \Delta = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$$

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For Linear Equations in Three Variables:

$$\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}, \Delta_1 = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}, \Delta_2 = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}, \Delta_3 = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}$$

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Methods of Solving Linear Equations in One Variable

Solving a linear equation with one variable is extremely easy and quick. To solve any two equations having only 1 variable, bring all the variable terms on one side and the constants on the other. The graphical method can also be used in which the point of intersection of the line with the x-axis or y-axis will give the solution of the equation.

For example, consider the equation $2x + 4 + 7 = 4x - 3 + x$

Here, combine the "x" terms and bring them on one side.

So,

$$5x - 2x = 14$$

$$\text{Or, } x = \frac{14}{3}$$

Methods of Solving Linear Equations in Two Variables

Any of the above-mentioned methods can be used i.e. graphical method, elimination method, substitution method, cross multiplication method, matrix method, determinants method.

Methods of Solving Linear Equations in Three or More Variables

For solving any equation having three or more variables, the graphical, elimination and the substitution method is not feasible. For three-variable equation, the cross-multiplication method is the most preferred method. Even matrix Cramer's rule is extremely useful for solving equations having 3 or more variables.

Frequently Asked Questions

What is a Linear Equation?

A linear equation is an equation where each variable has a degree of one. An example of a linear equation is $4x + 3y = 10$.

Example: 1 The sum of two numbers is 25. One of the numbers exceeds the other by 9. Find the numbers.

Solution:

Then the other number = $x + 9$

Let the number be x.

Sum of two numbers = 25

According to question, $x + x + 9 = 25$

$$\Rightarrow 2x + 9 = 25$$

$$\Rightarrow 2x = 25 - 9 \text{ (transposing 9 to the R.H.S changes to -9)}$$

$$\Rightarrow 2x = 16$$

$$\Rightarrow \frac{2x}{2} = \frac{16}{2} \text{ (divide by 2 on both the sides)}$$

$$\Rightarrow x = 8$$

$$\text{Therefore, } x + 9 = 8 + 9 = 17$$

Therefore, the two numbers are 8 and 17.

Example 2: The difference between the two numbers is 48. The ratio of the two numbers is 7:3. What are the two numbers?

Solution:

Let the common ratio be x .

Let the common ratio be x .

Their difference = 48

According to the question,

$$7x - 3x = 48$$

$$\Rightarrow 4x = 48$$

$$\Rightarrow x = \frac{48}{4}$$

$$\Rightarrow x = 12$$

$$\text{Therefore, } 7x = 7 \times 12 = 84$$

$$3x = 3 \times 12 = 36$$

Therefore, the two numbers are 84 and 36.