NCERT Class 11-Math'S: Chapter – 5 Complex Numbers and Quadratic Equations Part 1 (For CBSE, ICSE, IAS, NET, NRA 2022)

5.1 Overview

We know that the square of a real number is always non-negative e. g. $(4)^2 = 16$ and $(-4)^2 = 16$. Therefore, square root of 16 is ± 4 . What about the square root of a negative number? It is clear that a negative number cannot have a real square root. So we need to extend the system of real numbers to a system in which we can find out the square roots of negative numbers. Euler $(1707 - 1783)$ was the first mathematician to introduce the symbol i (iota) for positive square root of -1 i.e.. , $i = \sqrt{-1}$.

5.1. 1 Imaginary numbers

Square root of a negative number is called an imaginary number. , for example,

$$\sqrt{-9} = \sqrt{-1}\sqrt{9} = i3, \sqrt{-7} = \sqrt{-1}\sqrt{7} = i\sqrt{7}$$

5.1. 2 Integral powers of i

$$i = \sqrt{-1}, i^2 = -1, i^3 = i^2 i = -i, i^4 = \left(i^2\right)^2 = (-1)^2 = 1.$$

To compute i^n for $n > 4$, we divide n by 4 and write it in the form $n = 4m + r$, where m is quotient and r is remainder $(0 \leqslant r \leqslant 4)$

Hence $i^n = i^{4m+r} = \left(i^4\right)^m . (i)^r = (1)^m (i)^r = i^r$

For example, $(i)^{39} = i^{4\times 9+3} = \left(i^4\right)^9 .(i)^3 = -i$

And $(i)^{-435} = i^{-(4\times 108+3)} = (i)^{-(4\times 108)} .(i)^{-3}$

$$= \frac{1}{\left(i^4\right)^{108}} . \frac{1}{(i)^3} = \frac{i}{(i)^4} = i$$

(i) If a and b are positive real numbers, then

$$\sqrt{-a} \times \sqrt{-b} = \sqrt{-1}\sqrt{a} \times \sqrt{-1}\sqrt{b} = i\sqrt{a} \times i\sqrt{b} = -\sqrt{\mathrm{ab}}$$

(ii) $\sqrt{a}.\sqrt{b} = \sqrt{\text{ab}}$ if a and b are positive or at least one them is negative or zero.

However, $\sqrt{a}\sqrt{b} \neq \sqrt{\text{ab}}$ if a and b, both are negative.

5.1. 3 Complex numbers

(a) A number which can be written in the form $a + \text{ib}$, where a, b are real numbers and $i = \sqrt{-1}$ is called a complex number.

(b) If $z = a + \text{ib}$ is the complex number, then a and b are called real and imaginary parts, respectively, of the complex number and written as $\text{Re}\,(z) = a, \text{Im}\,(z) = b$.

(c) Order relations "greater than" and "less than" are not defined for complex numbers.

(d) If the imaginary part of a complex number is zero, then the complex number is known as purely real number and if real part is zero, then it is called purely imaginary number, for example, 2 is a purely real number because its imaginary part is zero and $3i$ is a purely imaginary number because its real part is zero.

5.1. 4 Algebra of complex numbers

(a) Two complex numbers $z_1 = a + \text{ib}$ and $z_2 = c + \text{id}$ are said to be equal if $a = c$ and $b = d$.

(b) Let $z_1 = a + \text{ib}$ and $z_2 = c + \text{id}$ be two complex numbers then $z_1 + z_2 = (a + c) + i\,(b + d)$.

5.1. 5 Addition of complex numbers satisfies the following properties

1. As the sum of two complex numbers is again a complex number, the set of complex numbers is closed with respect to addition.

2. Addition of complex numbers is commutative, i.e.. , $z_1 + z_2 = z_2 + z_1$

3. Addition of complex numbers is associative, i.e.. , $(z_1 + z_2) + z_3 = z_1 + (z_2 + z_3)$

4. For any complex number $z = x + i\ y$, there exist 0 , i.e.. , $(0 + 0i)$ complex number such that $z + 0 = 0 + z = z$, known as identity element for addition.

5. For any complex number $z = x + \text{iy}$, there always exists a number $-z = -a - \text{ib}$ such that $z + (-z) = (-z) + z = 0$ and is known as the additive inverse of z .

5.1. 6 Multiplication of complex numbers

Let $z_1 = a + \text{ib}$ and $z_2 = c + \text{id}$, be two complex numbers. Then $z_1 . z_2 = (a + \text{ib})\,(c + \text{id}) = (\text{ac} - \text{bd}) + i\,(\text{ad} + \text{bc})$

1. As the product of two complex numbers is a complex number, the set of complex numbers is closed with respect to multiplication.

2. Multiplication of complex numbers is commutative, i.e.. , $z_1 . z_2 = z_2 . z_1$

3. Multiplication of complex numbers is associative, i.e.. , $(z_1 . z_2) . z_3 = z_1 . (z_2 . z_3)$

4. For any complex number $z = x + \text{iy}$, there exists a complex number 1 , i.e.. , $(1 + 0i)$ such that $z . 1 = 1 . z = z$, known as identity element for multiplication.

5. For any non-zero complex number $z = x + i\ y$, there exists a complex number $\frac{1}{z}$ such that $z . \frac{1}{z} = \frac{1}{z} . z = 1, i.e.,$ multiplicative inverse of $a + \text{ib} = \frac{1}{a + \text{ib}} = \frac{a - \text{ib}}{a^2 + b^2}$.

6. For any three complex numbers z_1, z_2 and z_3 ,

$$z_1 . (z_2 + z_3) = z_1 . z_2 + z_1 . z_3$$

And $(z_1 + z_2) . z_3 = z_1 . z_3 + z_2 . z_3$

i.e.. , for complex numbers multiplication is distributive over addition.

5.1. 7 Let $z_1 = a + \text{ib}$ and $z_2\ (\neq 0) = c + \text{id}$. Then

$$z_1 \div z_2 = \frac{z_1}{z_2} = \frac{a + \text{ib}}{c + \text{id}} = \frac{(\text{ac} + \text{bd})}{c^2 + d^2} + i \frac{(\text{bc} - \text{ad})}{c^2 + d^2}$$

5.1. 8 Conjugate of a complex number

Let $z = a + \text{ib}$ be a complex number. Then a complex number obtained by changing the sign of imaginary part of the complex number is called the conjugate of z and it is denoted by $\overline{z}$, i.e.. , $\overline{z} = a - \text{ib}$.

Note that additive inverse of z is $-a - \text{ib}$ but conjugate of z is $a - \text{ib}$.

We have:

1. $\overline{(\overline{z})} = z$

2. $z + \overline{z} = 2\ \text{Re}(z), z - \overline{z} = 2i\ \text{Im}(z)$

3. $z = \overline{z}$, if z is purely real.

4. $z + \overline{z} = 0 \Longleftrightarrow z$ is purely imaginary

5. $z + \overline{z} = \{\text{Re}(z)\}^2 + \{\text{Im}(z)\}^2$

6. $\overline{(z_1 + z_2)} = \overline{z_1} + \overline{z_2}, \overline{(z_1 - z_2)} = \overline{z_1} - \overline{z_2}$

7. $\overline{(z_1 . z_2)} = (\overline{z_1}) + (\overline{z_2}), \overline{\left(\frac{z_1}{z_2}\right)} = \frac{(\overline{z_1})}{(\overline{z_2})} (\overline{z_2} \neq 0)$

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