NCERT Class 12-Mathematics: Exemplar Chapter – 7 Integrals Part 1 (For CBSE, ICSE, IAS, NET, NRA 2022)

7.1 Overview

7.1. 1

Let $\frac{d}{dx}F(x) = f(x)$. The, we write $\int f(x)\,dx = F(x) + C$. These integrals are called indefinite integrals or general integrals, C is called a constant of integration. All these integrals differ by a constant.

7.1. 2 If two functions differ by a constant, they have the same derivative.

7.1. 3 Geometrically, the statement $\int f(x)\,dx = F(x) + C = y$ (say) represents a family of curves. The different values of C correspond to different members of this family and these members can be obtained by shifting any one of the curves parallel to itself. Further, the tangents to the curves at the points of intersection of a line $x = a$ with the curves are parallel.

7.1. 4 Some properties of indefinite integrals

(i) The process of differentiation and integration are inverse of each other,

$i.e.,\ \frac{d}{dx}\int f(x)\,dx = F(x)$ and $\int f'(x)\,dx = F(x) + C$, where C is any arbitrary constant.

(ii) Two indefinite integrals with the same derivative lead to the same family of curves and so they are equivalent. So if f and g are two functions such that $\frac{d}{dx}\int f(x)\,dx = \frac{d}{dx}\int f(x)\,dx$, then $\int g(x)\,dx$ and $\int g(x)\,dx$ are equivalent.

(iii) The integral of the sum of two functions equals the sum of the integrals of the functions i.e.. ,
$\int (f(x) + g(x))\,dx = \int f(x)\,dx + \int g(x)\,dx$.

(iv) A constant factor may be written either before or after the integral sign, i.e.. ,

$\int a\,f(x)\,dx = a\int f(x)\,dx$, where $'a'$ is a constant.

(v) Properties (iii) and (iv) can be generalised to a finite number of functions $f_1, f_2, \ldots, f_n$ and the real numbers, $k_1, k_2, \ldots, k_n$ giving

$$\int \left((k_1 f_1(x) + k_2 f_2(x) + \ldots + k_n f_n(x)\right)\,dx = k_1\int f_1(x)\,dx + k_2\int f_2(x)\,dx + \ldots + k_n\int f_n(x)\,dx$$

7.1. 5 Methods of integration

There are some methods or techniques for finding the integral where we can-not directly select the ant derivative of function f by reducing them into standard forms. Some of these methods are based on

1. Integration by substitution
2. Integration using partial fractions
3. Integration by parts.

7.1. 6 Definite integral

The definite integral is denoted by $\int_a^b f(x)\,\mathrm{dx}$, where a is the lower limit of the integral and b is the upper limit of the integral. The definite integral is evaluated in the following two ways:

(i) The definite integral as the limit of the sum

(ii) $\int_a^b f(x)\,\mathrm{dx} = F(b) - F(a)$, if F is an ant derivative of $f(x)$.

7.1. 7 The definite integral as the limit of the sum

The definite integral $\int_a^b f(x)\,\mathrm{dx}$ is the area bounded by the curve $y = f(x)$, the ordinates $x = a, x = b$ and the x-axis and given by

$$\int_a^b f(x)\,\mathrm{dx} = (b-a)\lim_{n\to\infty}\frac{1}{n}[f(a) + f(a+h) + \ldots f(a+(n-1)h)]$$

$$\int_a^b f(x)\,\mathrm{dx} = \lim_{h\to 0} h[f(a) + f(a+h) + \ldots + f(a+(n-1)h)],$$

Where $h = \frac{b-a}{n} \to 0$ as $n \to \infty$

7.1. 8 Fundamental Theorem of Calculus

(i) Area function: The function $A(x)$ denotes the area function and is given by $A(x) = \int_a^x f(x)\,\mathrm{dx}$.

(ii) First Fundamental Theorem of integral Calculus

Let f be a continuous function on the closed interval $[a, b]$ and let $A(x)$ be the area function . Then $A'(x) = f(x)$ for all $x \in [a, b]$.

(iii) Second Fundamental Theorem of Integral Calculus

Let f be continuous function defined on the closed interval $[a, b]$ and F be an ant derivative of f

$$\int_a^b f(x)\,\mathrm{dx} = [F(x)]_a^b = F(b) - F(a).$$

7.1. 9 Some properties of Definite Integrals

$$P_0 : \int_a^b f(x)\,dx = \int_a^b f(t)\,dt$$

$$P_1 : \int_a^b f(x)\,dx = -\int_a^b f(x)\,dx, \text{ in particular, } \int_a^b f(x)\,dx = 0$$

$$P_2 : \int_a^b f(x)\,dx = -\int_a^b f(x)\,dx + \int_c^b f(x)\,dx$$

$$P_3 : \int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$$

$$P_5 : \int_0^{2a} f(x)\,dx = \int_0^b f(x)\,dx + \int_0^b f(2a-x) = f(x)$$

$$P_6 : \int_0^{2a} f(x)\,dx = \begin{cases} 2\int_0^{2a} f(x)\,dx, \text{if } (2a-x) = f(x), \\ 0, \text{if } f(2a-x) = -f(x) \end{cases}$$

$$P_7 : \int_{-a}^{a} f(x)\,dx = 2\int_0^a f(x)\,dx, \text{if } f \text{ is an even function } i.e., f(-x) = f(x)$$

(ii) $\int_0^a f(x)\,dx = 0, \text{if } f \text{ is an odd function } i.e., f(-x) =- f(x)$

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